

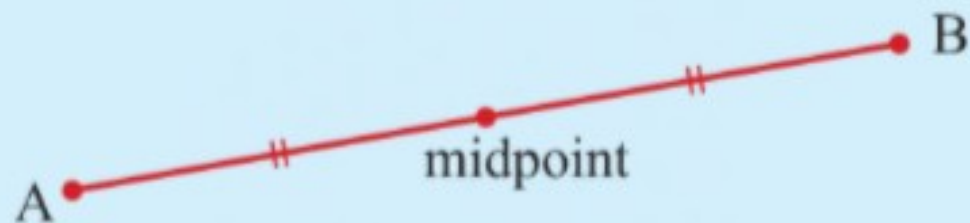
B

MIDPOINTS

B

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The **midpoint** of line segment $[AB]$ is the point which lies midway between points A and B.



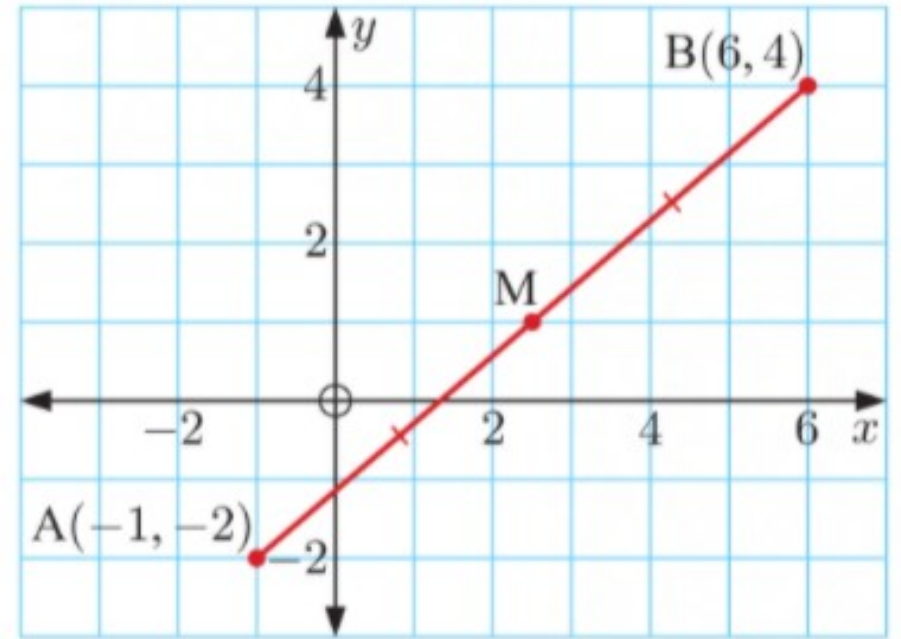
Consider the points $A(-1, -2)$ and $B(6, 4)$. From the diagram we see that the midpoint of $[AB]$ is $M(2\frac{1}{2}, 1)$.

The x -coordinate of M is the *average* of the x -coordinates of A and B .

$$\therefore \text{ the } x\text{-coordinate of } M = \frac{-1 + 6}{2} = \frac{5}{2} = 2\frac{1}{2}$$

The y -coordinate of M is the *average* of the y -coordinates of A and B .

$$\therefore \text{ the } y\text{-coordinate of } M = \frac{-2 + 4}{2} = 1$$



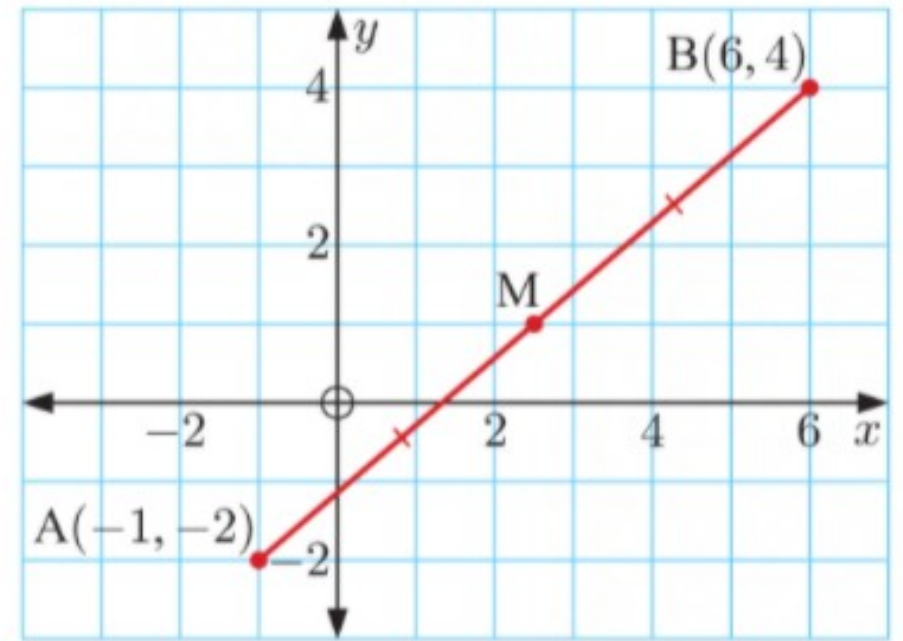
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If $A(x_1, y_1)$ and $B(x_2, y_2)$ are two points, then the **midpoint** of $[AB]$ has coordinates $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$.

Example 4

Find the midpoint of $[AB]$ given $A(-1, 3)$ and $B(4, 7)$.

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$A(-1, 3)$

$\uparrow \quad \uparrow$
 $x_1 \quad y_1$

$B(4, 7)$

$\uparrow \quad \uparrow$
 $x_2 \quad y_2$

The x -coordinate of the midpoint $= \frac{x_1 + x_2}{2} = \frac{-1 + 4}{2} = \frac{3}{2} = 1\frac{1}{2}$

The y -coordinate of the midpoint $= \frac{y_1 + y_2}{2} = \frac{3 + 7}{2} = 5$

So, the midpoint is $(1\frac{1}{2}, 5)$.

EXERCISE 6B

1 Find the coordinates of the midpoint of the line segment joining:

a $(8, 1)$ and $(2, 5)$

b $(2, -3)$ and $(0, 1)$

c $(3, 0)$ and $(0, 6)$

d $(-1, 4)$ and $(1, 4)$

e $(5, -3)$ and $(-1, 0)$

f $(5, 9)$ and $(-3, -4)$.

Example 5



M is the midpoint of $[AB]$. A is $(1, 3)$ and M is $(4, -2)$. Find the coordinates of B.

Example 5



M is the midpoint of [AB]. A is (1, 3) and M is (4, -2). Find the coordinates of B.

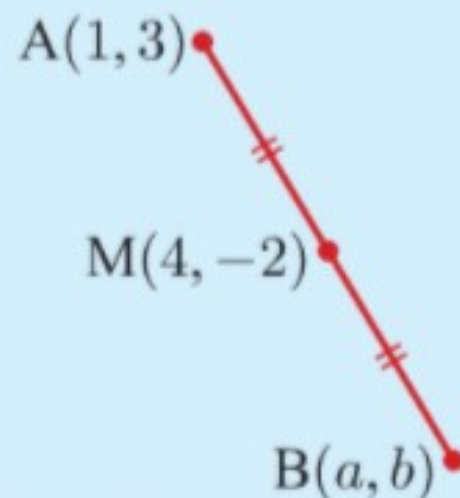
Suppose B has coordinates (a, b) .

$$\therefore \frac{a+1}{2} = 4 \quad \text{and} \quad \frac{b+3}{2} = -2$$

$$\therefore a+1 = 8 \quad \text{and} \quad b+3 = -4$$

$$\therefore a = 7 \quad \text{and} \quad b = -7$$

$$\therefore \text{B is } (7, -7).$$



Example 6

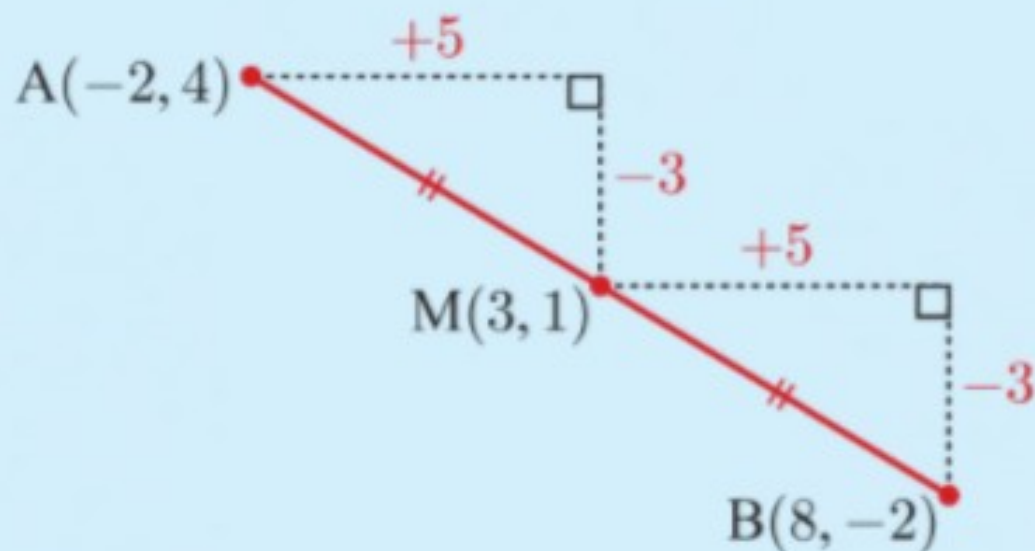
Self Tutor

Suppose A is $(-2, 4)$ and M is $(3, 1)$, where M is the midpoint of $[AB]$.
Use *equal steps* to find the coordinates of B.

$$x\text{-step: } -2 \xrightarrow{+5} 3 \xrightarrow{+5} 8$$

$$y\text{-step: } 4 \xrightarrow{-3} 1 \xrightarrow{-3} -2$$

$$\therefore B \text{ is } (8, -2).$$



2 M is the midpoint of [AB]. Find the coordinates of B for:

a $A(6, 4)$ and $M(3, -1)$

c $A(3, -2)$ and $M(1\frac{1}{2}, 2)$

e $A(7, -3)$ and $M(0, 0)$

b $A(-5, 0)$ and $M(0, -1)$

d $A(-1, -2)$ and $M(-\frac{1}{2}, 2\frac{1}{2})$

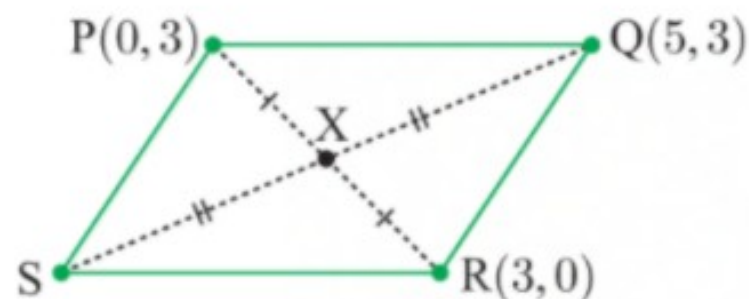
f $A(3, -1)$ and $M(0, -\frac{1}{2})$.

Check your answers using the *equal steps* method given in **Example 6**.

3 [AB] is a diameter of a circle with centre C. If A is $(3, -2)$ and B is $(-1, -4)$, find the coordinates of C.

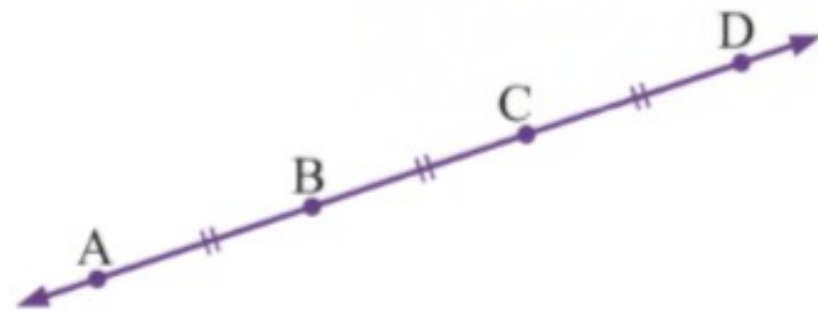
4 [PQ] is a diameter of a circle with centre $(3, -\frac{1}{2})$. If Q is $(-1, 2)$, find the coordinates of P.

5 The diagonals of parallelogram PQRS bisect each other at X. Find the coordinates of S.



6 Triangle ABC has vertices $A(-1, 3)$, $B(1, -1)$, and $C(5, 2)$. Find the length of the line segment from A to the midpoint of $[BC]$.

7 A, B, C, and D are four points on the same straight line. The distances between successive points are equal, as shown. If A is $(1, -3)$, C is $(4, a)$, and D is $(b, 5)$, find the values of a and b .



8 The midpoints of the sides of a triangle are $(5, 4)$, $(8, 5)$, and $(6, 0)$. Find the coordinates of the vertices of the triangle.

EXERCISE 6B

- 1** **a** $(5, 3)$ **b** $(1, -1)$ **c** $(\frac{3}{2}, 3)$ **d** $(0, 4)$
 e $(2, -\frac{3}{2})$ **f** $(1, \frac{5}{2})$
- 2** **a** $B(0, -6)$ **b** $B(5, -2)$ **c** $B(0, 6)$ **d** $B(0, 7)$
 e $B(-7, 3)$ **f** $B(-3, 0)$

3 $C(1, -3)$ **4** $P(7, -3)$ **5** $X(\frac{3}{2}, \frac{3}{2}), S(-2, 0)$

6 $\frac{\sqrt{89}}{2}$ units ≈ 4.72 units **7** $a = \frac{7}{3}, b = \frac{11}{2}$

8 Hint: Let the vertices be $A(a, b), B(c, d), C(e, f)$.

Use the midpoint formula to find six simple equations and solve them simultaneously.

Points are $(3, -1), (7, 9), (9, 1)$.